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ROYAL ASTRONOMICAL SOCIETY.

VOL. XIII. *April* 8, 1853. No. 6.

G. B. AIRY, Esq., President, in the Chair.

M. Littrow, Director of the Imperial Observatory at Vienna, was balloted for and unanimously elected an Associate of the Society.

On new Tables of the Moon's Parallax, to be substituted for those of Burckhardt. By J. C. Adams, Esq. M.A. F.R.S., &c. &c. (Appendix to the *Nautical Almanac* for 1856.)

(Read before the Royal Astronomical Society on the 14th of May, 1852.)

"The importance of an accurate knowledge of the moon's parallax is very evident. No observation of the moon's place can be compared with the tables, or turned to any practical use, without undergoing a preliminary reduction, of which the amount of the parallax is the most important element. Now the same theory by which the angular motion of the moon round the earth is determined gives likewise the form of the orbit, and therefore the proportion between the parallaxes at different times; hence, as the theory is sufficiently perfect to represent the place of the moon within 10″, it cannot be doubted that it would be competent to give the variations of the parallax within a small fraction of a second, provided the mean parallax were known. To determine this, however, by theory, it is necessary to know, in addition to the elements furnished by observations of the moon's motion, the ratio of the moon's mass to that of the earth. Hence, conversely, if the mean value of the parallax be deduced from corresponding observations of the moon's declination, made at distant points on the earth's surface, one means is afforded of finding the ratio of the masses.

"The most recent determination of the parallax by means of observations of this kind is contained in a paper by Mr. Henderson in the tenth volume of the *Memoirs of the Royal Astronomical Society*, and is founded on his own observations made at the Cape of Good Hope, combined with corresponding observations at Greenwich and Cambridge. In this paper Mr. Henderson compares the parallaxes deduced from observation with those calculated by means of the tables both of Burckhardt and Damoiseau. It is

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remarkable that he finds a difference of 1″·3 in the value of the mean parallax, according as one set of tables or the other is employed in the comparison, and not knowing which value to prefer, he adopts the mean of the two for his final result.

"If we consider, however, that the only part of this process which depends on the tables consists in the reduction of the actual parallaxes at the times of observation to the mean value, it is plain that so large a difference in the mean of thirty-four observations can only arise from intolerable errors in the periodic terms of parallax given by one of the two sets of tables.

"The parallax in Damoiseau's tables is given at once in the form in which it is furnished by theory, but that in Burckhardt's tables is adapted to his peculiar form of the arguments, and requires transformation in order to be compared with the former. When this was done, I found that several of the minor equations of parallax deduced from Burckhardt differed completely from their theoretical values given by Damoiseau.

"On further inquiry, I discovered that the difference between Burckhardt's equations of parallax and those of Bürg and Damoiseau had been long since remarked by Clausen, in a comparative analysis of the three sets of lunar tables given in the seventeenth volume of the *Astronomische Nachrichten*, but no notice appears to have been taken of this remark.

"With regard to the parallax, Burckhardt professes to have followed the theory of Laplace; but this agrees very closely with that of Damoiseau, so that errors have evidently been committed by him in the transformation of Laplace's formula.

"These appear to have originated in the following manner:—

"In the formation of Burckhardt's arguments of evection and variation, the *mean* longitude of the sun is employed. Now four of the errors in the coefficients of the minor equations may be accounted for, by supposing him to have erroneously employed the *true* instead of the *mean* longitude of the sun in forming the above-mentioned arguments. In another of these equations, the coefficient is taken with a wrong sign, and in another a wrong argument is employed.

"A strange fatality seems to have attended all Burckhardt's calculations respecting the moon's parallax. In the *Connaissance des Temps* for the year 15 of the Republic, he gives a comparison between the values furnished by Mayer's and Laplace's theories, and he concludes that the error of the former may sometimes amount to 7″.

"But this difference is caused almost wholly by an error in his own transformation of Laplace's expression. In the formation of Mayer's arguments of evection and variation, the *true* longitude of the sun is employed, but Burckhardt appears to have inadvertently used the *mean* longitude instead of it; an error which is the exact converse of the one above noticed with respect to his own tables.

"After examining Burckhardt's tables of parallax, I was naturally led to scrutinise more closely the results of the theories of

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Damoiseau, Plana, and Pontécoulant, with respect to the same subject. Although the differences between these were very trifling when compared with the errors of Burckhardt, still they were greater than we had a right to expect, considering the close agreement which existed with respect to the equations of longitude. In the theories of Damoiseau and Plana, the expression for the projection of the moon's radius vector on the ecliptic in terms of her true longitude is required in order to find the relation between that longitude and *the time*, and therefore no pains have been spared to obtain it with accuracy; but in the subsequent operations and transformations necessary in order to deduce the expression for the parallax in terms of the time, the same care has not been employed. In Pontécoulant's theory the time is taken as the independent variable, and, consequently, the analytical expression for the parallax in the form required is obtained immediately, and is developed to as great an extent as the corresponding expression for the longitude; yet in the conversion of his formula into numbers he neglects all the terms beyond the fifth order, so that several of the resulting coefficients are sensibly in error.

"I have endeavoured to supply these defects and omissions.

"In the seventeenth volume of the *Astronomische Nachrichten*, M. Hansen gives the expression which he has obtained for the logarithm of the sine of the horizontal parallax, by means of his new method of treating the lunar theory. I have transformed this expression with the care which its great value deserves, so as to compare it with the results of the former theories.

"The agreement thus found between the several theories is most satisfactory, the difference of the separate values of each coefficient and the general mean rarely amounting to a hundredth of a second. There are only two instances in which this amount is much exceeded. One of these relates to the constant of parallax, the value of which, given by M. Hansen's method, is 0″·06 less than the corresponding value found from the same fundamental data by the other methods, and the second relates to the term whose argument in Damoiseau's notation is $t+z$, the coefficient being 0″·146 according to Damoiseau and Plana, 0″·140 according to Pontécoulant, and 0″·181 according to Hansen.

"The values of the constant of parallax which I have deduced from the theories of Damoiseau, Plana, and Pontécoulant, agree perfectly with one another, and from the particular examination which I have given to this subject, I am induced to place considerable reliance on the result. It is possible that M. Hansen's definitive value of the constant may differ slightly from that which he has given in the paper above referred to.

"From the value of the constant of nutation found by M. Peters, it follows that the ratio of the moon's mass to that of the earth is as 1 to 81·5 nearly. Employing this ratio, together with the dimensions of the earth according to Bessel, and the length of the seconds pendulum in latitude 35°¼, deduced from Mr. Baily's Report on Foster's pendulum experiments, I find the value of the constant of parallax to be 3422″·325.

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"Now Henderson, in the paper cited above, has found the value of the constant, by comparison with Damoiseau's table, to be 3422″·46.

"It should, however, be remarked, that what the tables call the parallax is more strictly the *sine* of the parallax converted into seconds of arc. In Henderson's calculations he has taken the tabular quantity to denote the parallax itself, so that the value found must be diminished by 0″·15, in order to obtain the constant of the *sine* of the parallax. Thus the value deduced in this manner is 3422″·31, a result admirably agreeing with that just derived from theory.

"I have carefully transformed the expression for the parallax given by theory, so as to make it depend on Burckhardt's arguments of longitude, and from the resulting formula Mr. Farley has calculated the tables which are appended to this paper. Constants are added to the several equations so as to render them always positive.

"The minor equations of equatoreal horizontal parallax are comprised in Table I.;

"Table II. contains the equation depending on the argument of evection;

"Table III. that depending on the argument of variation; and

"Table IV. that depending on the argument of anomaly.

"The formulæ employed in their construction are the following, in which

E denotes Burckhardt's argument of evection;
V that of variation; and
A that of anomaly;

and the arguments of the minor equations are denoted by their numbers as in Burckhardt.

$$
\begin{array}{rcl}
{}'' & & {}'' \\
0{\cdot}34 & - & 0{\cdot}34 \cos (\text{Arg. } 1) \\
1{\cdot}73 & + & 1{\cdot}73 \cos (\text{Arg. } 2) \\
1{\cdot}46 & + & 1{\cdot}46 \cos (\text{Arg. } 4) \\
0{\cdot}87 & + & 0{\cdot}87 \cos (\text{Arg. } 5) \\
0{\cdot}71 & - & 0{\cdot}71 \cos (\text{Arg. } 6) \\
0{\cdot}11 & - & 0{\cdot}11 \cos (\text{Arg. } 7) \\
0{\cdot}62 & - & 0{\cdot}62 \cos (\text{Arg. } 8) \\
1{\cdot}81 & - & 0{\cdot}05 \cos (\text{Arg. } 9) \\
0{\cdot}21 & - & 0{\cdot}21 \cos (\text{Arg. } 12) \\
0{\cdot}16 & - & 0{\cdot}16 \cos (\text{Arg. } 13) \\
0{\cdot}14 & + & 0{\cdot}14 \cos (\text{Arg. } 16) \\
0{\cdot}12 & + & 0{\cdot}12 \cos (\text{Arg. } 23) \\
0{\cdot}10 & + & 0{\cdot}10 \cos (\text{Arg. } 25) \\
36{\cdot}81 & + & 37{\cdot}22 \cos E + 0''{\cdot}41 \cos 2E \\
26{\cdot}18 & - & 0{\cdot}94 \cos V + 26''{\cdot}34 \cos 2V + 0''{\cdot}16 \cos 4V \\
55'\,50{\cdot}92 & + & 187{\cdot}14 \cos A + 10''{\cdot}27 \cos 2A + 0''{\cdot}64 \cos 3A + 0''{\cdot}04 \cos 4A
\end{array}
$$

"In this formula, a few terms have been neglected, the largest of the coefficients of which does not exceed 0″·08.

"The sum of the constants in this formula is 3422″·29, slightly

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differing from what is called the constant of parallax, in consequence of the change in the form of developement.

“For the sake of comparison I will here give the formula on which Burckhardt's own tables are constructed, which is as follows:—

$$\begin{array}{rcl}
'' & & '' \\
0\cdot4 & - & 0\cdot4 \cos (\text{Arg. } 1) \\
0\cdot8 & + & 0\cdot8 \cos (\text{Arg. } 2) \\
0\cdot3 & + & 0\cdot3 \cos (\text{Arg. } 4) \\
0\cdot8 & + & 0\cdot8 \cos (\text{Arg. } 5) \\
1\cdot1 & + & 0\cdot8 \cos (\text{Arg. } 6) \\
0\cdot6 & - & 0\cdot6 \cos (\text{Arg. } 8) \\
1\cdot8 & + & 1\cdot8 \cos 2 (\text{Arg. } 9) \\
0\cdot7 & + & 0\cdot7 \cos (\text{Arg. } 12) \\
1\cdot0 & + & 1\cdot0 \cos (\text{Arg. } 13) \\
43\cdot0 & + & 37\cdot4 \cos E + 0''\cdot4 \cos 2 E \\
30\cdot0 & - & 1\cdot0 \cos V + 26''\cdot3 \cos 2 V + 0''\cdot3 \cos 3 V \\
55' \, 40\cdot0 & + & 187\cdot0 \cos A + 10''\cdot2 \cos 2 A + 0''\cdot5 \cos 3 A
\end{array}$$

“The sum of the constants in this formula is 3420″·5.

“The errors of the coefficients of equations 2 and 12 arise from the mistake respecting the formation of the argument of variation before explained, and those of the coefficients of equations 4 and 13 from the similar mistake respecting the argument of evection.

“Equation 6 is taken with a wrong sign, and in the variation equation 3 V appears to be wrongly substituted for 4 V, though I find that the corresponding term, when reduced to Burckhardt's form, has a smaller coefficient.

“In consequence of the way in which most of these errors originate, their amount will be generally greatest in March and September, and least about the beginning of January and July, when the sun's mean and true places coincide.

“The total error of Burckhardt's tables may amount to nearly 6″, independently of the change in the value of the constant.

“Looking at the accuracy of modern observations, it is easy to imagine to what an extent the value of comparisons between observed and tabular places may be diminished by their being liable to an error of this kind.

“In determining differences of longitude by means of occultations, it is plain that the results may be considerably affected by such an error in the parallax. It has often been remarked, that differences of longitude obtained by means of different occultations are not so consistent with each other as might be expected from the precise character of the observation, and I have no doubt that a great part of the discrepancy is to be attributed to the use of an erroneous parallax.

“Mr. Maclear's observations at the Cape, combined with European observations, would doubtless furnish most valuable materials for a new determination of the constant of parallax, care being of course taken to employ correct tables in the reductions; and such a work would be a useful contribution to astronomy.

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"In order to facilitate these and similar objects, Mr. Stratford has calculated the parallaxes from my tables for each Greenwich mean noon in the years 1840–1855, and has thus obtained the corrections to be applied to the corresponding quantities given in the *Nautical Almanac.*

"These corrections are embodied in tables which are appended to the present paper. Subsequently to 1855, the moon's parallax given in the *Nautical Almanac* is calculated from my tables."

Having made these preliminary remarks, Mr. Adams proceeds to give his new tables of the moon's parallax. These are succeeded by the corrections to be applied to the calculated parallaxes in the *Nautical Almanac* for the years 1840–55, as above referred to. The following are the corrections for the year 1853:—

Day of Mth.	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
	″	″	″	″	″	″	″	″	″	″	″	″
1	−0·4	−1·1	+0·5	−0·7	−0·4	−0·8	−0·2	+0·2	−1·0	−2·5	−3·8	−2·8
2	0·3	2·1	−0·6	1·7	1·1	0·6	0·0	0·0	2·0	3·6	4·9	3·2
3	0·4	2·9	1·7	2·5	1·5	0·4	+0·1	−0·4	3·2	5·0	5·5	3·4
4	0·8	3·8	3·0	3·1	1·8	−0·2	0·3	0·9	4·4	5·9	5·6	3·5
5	1·3	4·4	4·0	3·4	1·7	+0·3	0·5	1·5	5·0	6·4	5·5	3·2
6	1·8	4·5	4·7	3·3	1·3	0·5	0·5	2·3	5·4	6·3	4·7	2·8
7	2·3	4·4	4·6	2·9	0·8	0·8	+0·1	2·8	5·1	5·8	4·3	2·3
8	2·8	3·9	4·3	2·4	−0·4	0·9	−0·2	2·9	4·4	5·1	3·5	1·7
9	3·0	3·3	3·7	1·9	0·0	0·7	0·5	2·7	3·3	4·0	2·3	1·0
10	3·1	2·7	3·0	1·2	+0·2	0·6	0·6	2·3	2·5	3·1	1·2	0·3
11	3·0	2·1	2·4	0·9	0·4	+0·4	0·9	1·6	1·6	2·0	−0·5	−0·2
12	2·8	1·8	1·9	0·7	+0·2	−0·2	0·9	1·0	−0·8	−0·7	+0·2	+0·2
13	2·5	1·3	1·4	0·5	−0·1	0·8	1·0	0·8	+0·1	+0·2	0·9	0·5
14	2·2	1·0	1·0	0·7	0·6	1·6	1·0	0·4	0·7	1·3	1·3	0·6
15	1·8	0·9	0·8	1·0	1·5	2·4	1·2	0·3	1·4	2·0	1·3	0·5
16	1·5	1·0	0·7	1·7	2·5	3·0	1·5	−0·2	1·8	2·4	1·1	+0·1
17	1·5	1·0	1·0	2·6	3·4	3·2	1·6	+0·1	1·9	2·3	+0·5	−0·2
18	1·4	1·1	1·4	3·3	3·9	3·2	1·7	+0·1	1·7	1·7	0·0	0·6
19	1·3	0·8	1·6	3·4	4·2	3·0	1·6	0·0	1·2	1·2	−0·3	1·2
20	1·0	−0·4	1·8	3·3	3·7	2·8	1·8	−0·5	0·4	0·6	0·4	1·6
21	−0·6	+0·6	1·7	2·5	2·9	2·7	2·1	0·8	0·1	0·3	0·3	1·9
22	+0·1	1·5	1·3	1·2	2·1	2·7	2·3	1·0	0·0	0·5	0·3	1·9
23	0·8	2·1	−0·1	−0·1	1·4	2·8	2·6	1·2	0·2	0·6	0·2	1·9
24	1·4	2·6	+1·0	+0·7	1·2	2·8	2·4	1·0	0·7	1·0	0·1	1·9
25	1·9	2·7	1·9	1·2	1·4	2·5	2·2	0·7	1·1	1·2	0·3	2·1
26	1·9	2·5	2·5	1·2	1·1	2·0	1·7	−0·1	1·2	1·2	0·4	2·1
27	1·6	2·0	2·7	1·0	0·8	1·6	1·1	+0·4	1·1	0·9	1·0	2·1
28	1·2	+1·5	2·5	0·8	0·4	1·1	0·6	0·6	+0·9	+0·3	1·4	1·9
29	0·6	...	2·1	0·6	0·4	0·8	−0·2	0·7	0·0	−0·8	1·7	1·9
30	+0·1	...	1·3	+0·1	0·5	−0·6	+0·3	+0·5	−1·2	1·7	−2·2	1·8
31	−0·5	...	+0·4	...	−0·7	...	+0·2	−0·2	...	−2·8	...	−1·8